To the Editor

The vacancy solution model (VSM) of Danner and coworkers (1980, 1985) for adsorption of gas mixtures contains an error. At the limit of zero pressure, adsorbate-adsorbate interactions on the surface of the adsorbent are negligible and the specific amount adsorbed of any species (n_i) is given by Henry's law:

$$n_i = \frac{B_{is} P y_i}{RT} \tag{1}$$

where y_i is mole fraction of i in the gas phase, and B_{is} is the adsorption second virial coefficient for the interaction between the ith species and the surface, obtainable from the pure-gas isotherm by:

$$\lim_{P\to 0} \frac{dn}{dP} = \lim_{P\to 0} \frac{n}{P} = \frac{B_{1s}}{RT} \tag{2}$$

It follows that the limiting value of selectivity of i relative to j ($s_{i,j}$) is given by the ratio of virial coefficients:

$$\lim_{P\to 0} s_{i,j} = \lim_{P\to 0} \frac{n_i/y_i}{n_j/y_j} = \frac{B_{is}}{B_{js}} \tag{3}$$

Selectivities measured at different gas-phase compositions should all extrapolate to the same limit (Henry's law), Figure 1. VSM violates Henry's law.

In the version of VSM by Cochran, Kabel and Danner (1985), the fugacity of ith component is given by the equation:

$$Py_i\phi_i = \frac{\gamma_i^s x_i n_m n_i^\infty \exp(\alpha_{iv})}{n_m^\infty b_i (1+\alpha_{iv})} \times \exp\left\{\left[\left(\frac{n_i^\infty - n_m^\infty}{n_m}\right) - 1\right] \ln(\gamma_v^s x_v^s)\right\} \tag{4}$$

The notation is the same as that in the original paper (1985). However, that paper contains the typographical error that the factor $\{[(n_i^\infty - n_m^\infty)/n_m] - 1\} \ln(\gamma_v^s x_v^s)$ is not exponentiated; this was noted in an Erratum (1985) and corrected in the later paper of High and Danner (1986).

From Eq. 4, the selectivity $s_{1,2}$ for a perfect ($\phi_i \to 1$ as $P \to 0$) binary gas mixture is given by the ratio:

$$s_{1,2} = \frac{x_1/y_1}{x_2/y_2} = \frac{b_1\gamma_2^s n_2^\infty \exp(\alpha_{2v})(1+\alpha_{1v})}{b_2\gamma_1^s n_1^\infty \exp(\alpha_{1v})(1+\alpha_{2v})} \times \exp\left[\left(\frac{n_2^\infty - n_1^\infty}{n_m}\right) \ln(\gamma_v^s x_v^s)\right] \tag{5}$$

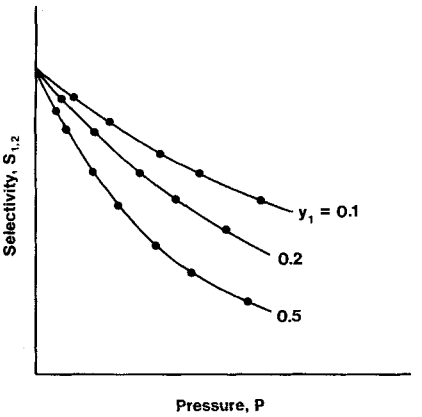


Figure 1. Selectivity $s_{1,2}$ of a binary mixture at constant T. Curves for different values of y_1 must intersect at $P = 0$.

In the limit as $P \to 0$: $n_m \to 0$, $x_1^s \to 0$, $x_2^s \to 0$, $x_v^s \to 1$, and $\gamma_v^s \to 1$. Also:

$$\lim_{P\to 0} \frac{\gamma_2^s}{\gamma_1^s} = \frac{\exp(\alpha_{1v})(1+\alpha_{2v})}{\exp(\alpha_{2v})(1+\alpha_{1v})} \tag{6}$$

Therefore, the limiting value of selectivity is:

$$\lim_{P\to 0} s_{1,2} = \frac{b_1 n_2^\infty}{b_2 n_1^\infty} \times \exp\left[\left(\frac{n_2^\infty - n_1^\infty}{n_m}\right) \ln(\gamma_v^s x_v^s)\right] \tag{7}$$

Application of L'Hospital's rule yields the limit:

$$\lim_{P\to 0}\left[\frac{\ln(\gamma_v^s x_v^s)}{n_m}\right] = \frac{\lim_{P\to 0}\left(\frac{d\ln(x_v^s)}{dn_m}\right)}{\lim_{P\to 0}\left(\frac{dn_m}{dn_m}\right)} = \frac{\lim_{P\to 0}\left(\frac{-1}{x_v^s n_m^\infty}\right)}{1} = -\frac{1}{n_m^\infty} \tag{8}$$

and Eq. 7 reduces to:

$$\lim_{P\to 0} s_{1,2} = \frac{b_1 n_2^\infty}{b_2 n_1^\infty} \times \exp\left(\frac{n_1^\infty - n_2^\infty}{x_1 n_1^\infty + x_2 n_2^\infty}\right) \tag{9}$$

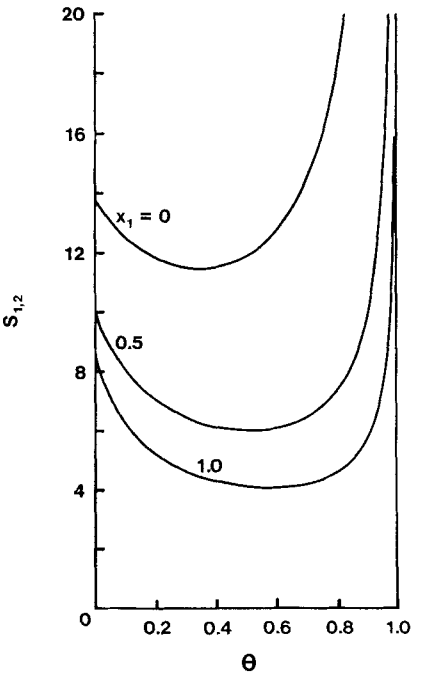


Figure 2. Selectivity $s_{1,2}$ of a binary mixture calculated by Eq. 5 (vacancy solution model) as a function of θ for different values of adsorbed-phase composition x_1. Values $s_{1,2}$ do not intersect at $\theta = 0$.

n_1^∞ and n_2^∞ are the saturation capacities of the adsorbent for component nos. 1 and 2, respectively. Except for the special case where $n_1^\infty = n_2^\infty$, Eq. 9 for VSM predicts that the limiting value of selectivity is a function of composition. This contradicts Eq. 3. Equation 9 is also obtained for other forms of VSM such as that of Suwanayuen and Danner (1980).

Figure 2 shows a plot of selectivity ($s_{1,2}$) from VSM as a function of the fractional coverage of the surface $\theta = n_m/n_m^\infty$ for values of adsorbed-phase composition varying from 0 to 1 (cf. Figure 1). Selectivity was calculated from Eq. 5 using the constants in Table 1. Since the curves on

Table 1. Constants for Eq. 5

Component No.	n_i^∞	b_i	α_{iv}
1	2	10	3
2	1	1	1

Figure 2 do not intersect at $\theta = 0$, they cannot represent the behavior of real systems.

Violation of Henry's law by VSM places it at a disadvantage with respect to other theories that satisfy Eq. 3, e.g., that of Ruthven, Loughlin and Holborow (1973).

Literature cited

Cochran, T. W., R. L. Kabel, and R. P. Danner, "Vacancy Solution Theory of Adsorption Using Flory-Huggins Activity Coefficient Equations," *AIChE J.*, **31**, 268 (1985).

Erratum, *AIChE J.*, **31**, 2082 (1985).

High, M. S., and R. P. Danner, "Treatment of Gas-Solid Adsorption Data by the Error-in-Variables Method," *AIChE J.*, **32**, 1138 (1986).

Ruthven, D. M., K. F. Loughlin, and K. A. Holborow, "Multicomponent sorption equilibrium in molecular sieve zeolites," *Chem. Eng. Sci.*, **28**, 701 (1973).

Suwanayuen, S., and R. P. Danner, "Vacancy Solution Theory of Adsorption from Gas Mixtures," *AIChE J.*, **26**, 76 (1980).

Orhan Talu
Dept. of Chemical Engineering
Cleveland State University
Cleveland, OH 44115

Alan L. Myers
Dept. of Chemical Engineering
University of Pennsylvania
Philadelphia, PA 19104

The contents of the above letter were originally included in the paper by the same authors appearing elsewhere in this issue. They were published separately as a Letter to the Editor at the Editor's request.

To the Editor:

Ramachandran and Smith (1979) showed that in principle the adsorption equilibrium constant K for gaseous adsorbates in trickle-bed reactors could be obtained from experimentally measured zero and first moments of the effluent gas stream, provided that the adsorption was linear. This is an attractive possibility for measuring K, since both moments m_0 and m_1 can be evaluated from the same dynamic response experiment. Sensitivity analysis of the equations, however, shows that a small uncertainty in m_0 and m_1 leads to large errors in K. An uncertainty of $\pm 10\%$ in m_0 can give errors in K up to 120%.

We have investigated this observation with experimental pulse-response measurements for adsorption of SO_2 from He carrier gas onto activated carbon in a trickle bed with water as the liquid. For this system at 298 K, the adsorption equilibrium constant K is 30×10^{-3} m^3/kg (Komiyama and Smith, 1975). Our experimental results, a set of six different experimental conditions, allowed us to calculate an average K value of 37 $(\pm 7) \times 10^{-3}$ m^3/kg. The large random errors associated with the response measurements call into question the calculated value of K.

Unless very accurate moment values can be determined, results for K by this dynamic method will be uncertain and questionable. Accurate moments for the bed of adsorbent particles are difficult to obtain because significant mass transfer can occur in the regions before and after the bed. Such end effects must be accounted for in calculating the moments for the bed alone. When end effects are included, certain information can be obtained from the zero moments of dynamic response experiments. However, first moments, more sensitive to random errors in the trickle-bed reactor, must be used with caution.

Literature cited

Ramachandran, P. A., and J. M. Smith, "Dynamic Behavior of Trickle-Bed Reactors," *Chem. Eng. Sci.*, **34**, 75 (1979).

Komiyama, H., and J. M. Smith, "Adsorption and Reaction of Sulfur Dioxide in Slurries of Activated Carbon," *AIChE J.*, **21**, 664 (1975).

J. F. Gabitto
B. J. McCoy
Dept. of Chemical Engineering
University of California
Davis, CA 95616

To the Editor:

In the paper entitled "The Continuous–Flow Gravity Thickener: Steady-State Behavior" (Feb., 1988, p. 239), White, Landman, White, and Buscall take a fresh mathematical approach to steady-state thickening. Most of the assumptions and conclusions they reach turn out to be not fundamentally different from those already in the literature. Some of their conclusions, however, derive from misinterpretation of their mathematical model and would be misleading.

They conclude that, to attain any given underflow concentration ϕ_u in diverging thickeners, "the corresponding bed height is smaller than for cylindrical and converging thickeners." This comparison, however, is based on having the same thickener radius at the top of the compression zone for each case. In the case of diverging thickeners, the cross-sectional area, A_z, at any depth z is greater than that at the top of the compression zone. Where the area is greater, the solids flux through the level in a continuous thickener is less than that through a cylindrical or converging thickener at the same level. Therefore, the distance, dz, which takes to produce a given increment $d\phi_z$, is less. (See below.) Under such a comparison, it would indeed follow that the bed height in a diverging thickener would be less than that in a cylindrical or converging one. But the thickener would have a larger maximum diameter and would be a bigger thickener.

If the comparison is made among thickeners with the same maximum areas, the area A_z at any other level z would be lower for both converging and diverging thickeners than for cylindrical ones. Therefore, the distance, dz, needed to produce a given increment to any given concentration would be greater in both the noncylindrical ones. Thus, the total height needed to develop a given total concentration difference (i.e., from ϕ_o to ϕ_u) would be greater.

Although the comparison made in the paper is mathematically valid, it is meaningless (or naive) from a technological standpoint. It would be easier and usually cheaper to build a cylindrical thickener than one of the same diameter with tumble-home sides, and the cylindrical one would be more effective. There is no basis for concluding that reducing the diameter of a thickener at the top, as is shown in Figure 2 of the paper, would increase its capacity.

The authors also conclude that there is no limit to the underflow concentration that can be attained with a given solids throughput in a diverging thickener. This is obviously wrong. It follows their mathematical model, but is a case of misapplication. The constitutive relationship they assume for connecting network stress to local concentration entails that the network be infinitely compressible. Such is not the case. Their assumed relationship

is not valid for high stresses. (What is true is that there would be no limit to the network stress attainable in an unboundedly diverging thickener.)

The authors further conclude that there is no limit to the amount of solids that can be fed through a (boundlessly) diverging thickener, while still maintaining a given underflow concentration. This is true enough. But, with very high solids throughputs, the ultimate diameter needed at the bottom of the thickener would be correspondingly very large. If no limit is placed on the size of the thickener, obviously there is no limit to the amount of solids it will handle. In such a case, the narrow top portion would act as little more than a conduit to pass the feed to the greatly expanded regions far below. If, on the other hand, the area at the bottom of the thickener is limited or held to a fixed value, there would indeed be a limit to its flux-handling capacity, just as there is for cylindrical and converging ones. (These limitations form the standard basis for thickener design.)

After considerable discussion, the authors accept the assumption that the solids pressure at any point will be essentially equal to the yield value of the solids structure in the pulp: $p = P_y(\phi)$. They also accept that the hydraulic conductivity of the solids structure will be a function of concentration: $K = K(\phi)$. These are the conventional assumptions, made intuitively without the mathematical justifications advanced by the authors, by almost everyone since introduced by Michaels and Bolger (1962). Under these assumptions, it is easily shown (Fitch, 1966, 1979) that:

$$(dz/d\phi) = (dz/d\phi)^*[S/(S - G)] \quad (1)$$

where

$(dz/d\phi)^*$ = solids concentration gradient attained after subsidence is complete
S = Kynch settling flux in the absence of solids pressure gradient
G = actual settling flux

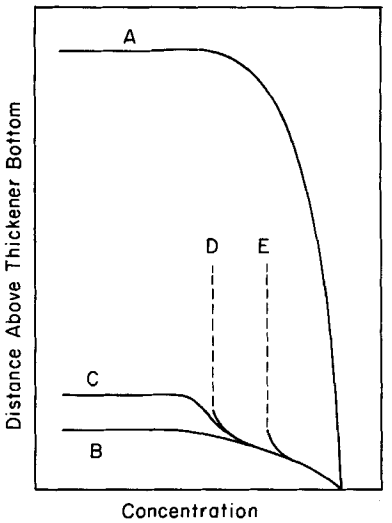
All being functions of concentration ϕ.

The term $[S/(S - G)]$ can be considered a "stretch factor," that determines how much larger a height increment dz must be in an actively compressing bed than it would be in a fully compressed one to obtain a given concentration increment $d\phi$. And the smaller G is at any given concentration, the smaller the stretch factor. But if G becomes equal to S at any concentration in or out of the compression regime, the stretch factor becomes infinite and the specified thickener is too small to handle the specified throughput while producing the specified underflow concentration.

In a cylindrical thickener $G = G_t(1 - \phi/\phi_u)$, where G_t is the total solids flux through the thickener. More generally, and thus for noncylindrical ones, G would equal $(Q_u/A_z)(1 - \phi/\phi_u)$. The larger A_z, the smaller the actual settling flux G, and the smaller the stretch factor.

Since A_z is a function of z, and all the rest of the variables are functions of ϕ, these functions can be substituted in Eq. 1, which can then be solved numerically to obtain concentration profiles. Calculated profiles for a hypothetical case are shown in Figure 1. As will be seen, the profiles for both diverging and converging designs run above that for the cylindrical thickener.

The significance of the profiles for diverging thickeners shown in Figure 1 may not be obvious without explanation of how they are obtained. In order to calculate profiles for conical thickeners, it turns out to be mathematically expedient to integrate numerically from ϕ_u at the bottom of the thickener to ϕ_o at the top of the compression zone. By this procedure, the maximum thickener area at the top of the compression zone is free to reach whatever value is necessary to meet the specified solids throughput and underflow concentration. There are no pinch points, and there is always a solution. When the same computer program is used to calculate profiles for a cylindrical or a diverging thickener, the maximum area is specified and is not free to come to whatever value is necessary. But, if G becomes equal to S at any concentration, the calculated stretch factor, and hence $dz/d\phi$, will become infinite. This will show that no solution is possible. The thickener is just too small to meet the specified throughput and underflow concentration.



A: cone thickener, 30° side slope
B: cylindrical thickener
C: diverging thickener, 6.2° side slope
D: diverging thickener, 6.5° side slope
E: diverging thickener, 30° side slope

Figure 1. Concentration profiles.

The calculation procedure is then to first determine the profile for a conical thickener with a specified solids throughput rate and underflow concentration. The solution will show the thickener diameter needed, namely that at the top of the compression zone. Then a profile for the cylindrical thickener, having the same diameter, is calculated. It shows no pinch point, and a concentration profile far below that for the conical thickener. But the profiles calculated for diverging thickeners having a wall slope of more than 6.2 degrees do show pinch points. When the walls slope inward more than this, the area at some level becomes critical. Such thickeners would have to be made larger to handle the load. Clearly, then, sloping the walls inward decreases the effectiveness of a thickener.

Literature cited

Fitch, B., "Current Theory and Thickener Design," *Ind. & Eng. Chem.*, **58**, 18 (1966).

Fitch, B., "Sedimentation of Flocculent Suspensions, State of the Art," *AIChE J.*, **25**, 913, (Dec., 1979).

Landman, K. A., L. R. White, and R. Buscall, "The Continuous-Flow Gravity Thickener: Steady State Behavior," *AIChE J.*, **34**, 239 (Feb., 1988).

Michaels, A. S., and J. C. Bolger, "Settling Rates and Sediment Volumes of Flocculated Kaolin Suspensions," *I. & E. C. Fund.*, **1**, 24 (1962).

Bryant Fitch
1570 Oak Street
Napa, CA 94559

Reply:

Fitch has not, we think, fully appreciated the rationale for this work. In our reading of the thickener literature, there does not appear to have been a satisfactory discussion of the role of the rheology

of the suspension in the thickening process. We tried to redress this situation by establishing that the usual constitutive assumption that the network pressure is a function of volume fraction (or voidage), and volume fraction only, is valid only when certain kinetic constraints are obeyed. In particular, when the collapse process itself is not rate-limiting, the network pressure stays close to the network yield-stress, this being regarded as a function only of concentration. In this way, we have given the standard empirical constitutive assumption a sound physical basis and have provided a quantitative criterion for its applicability.

We do not mean to suggest that there is no limit to the underflow concentration that can be achieved in a diverging thickener. Clearly there must be since the yield stress has to diverge as the particles close-pack. This limit was not included in our constitutive law and thus our remark about there being no limit should not be taken to apply outside the concentration range over which the suspension is compressible. With hindsight, it may, however, have been helpful to have been explicit on this point.

We refute the suggestion that we have misinterpreted our model, although it would appear that we have misled Fitch. The apparent conflict between what is shown by our Figure 8 and what Fitch shows in his Figure 1 arises from a difference in what is held constant in the comparison of parallel and nonparallel thickeners. We have chosen to fix the underflow concentration and the flux per unit area at the top, whereas Fitch has chosen to fix the underflow concentration and the flux per unit area at the bottom. Under the latter circumstance, a solution in terms of the required height of the conical thickener can always be found, whereas in the former case there is a limiting value of the underflow concentration above which no solution exists. There is, however, no inconsistency between these two observations and from the point of view of a fundamental study there would seem to be no reason to prefer the use of one point of reference to the other. Indeed when it comes to practice, it is not clear to us that specifying the flux per unit area at the bottom is the appropriate way to approach the design of a *conical* thickener because it places no constraint on the maximum diameter. We thus emphasize that there is no conflict between our conclusions and those of Fitch, there is no misinterpretation, and the choice of the point of reference would appear to us to be a matter only of convenience.

Although it was not part of our brief to consider the economics of thickener design, Fitch may well be correct in saying that the diverging thickener would be more expensive than a cylindrical one for a given set of input parameters. There may, however, be good reasons for choosing this shape in certain circumstances—for example, to avoid hold-up on the walls in the case of sediments with large yield stresses in shear and the reduction in effective cross-section that this implies. This is, however, only a tentative suggestion and we leave it to the practitioners to decide upon its worth. We do not, for example, suggest what is shown in Figure 2 as an optimum profile as Fitch seems to think.

Richard Buscall
ICI PLC
The Heath, Runcorn
Cheshire, England

Lee White
Dept. of Mathematics
University of Melbourne
Parkville Vic 3052
Australia

To the Editor:

Landman, White, and Buscall (Feb., 1988, p. 239) considered the capacity not only of cylindrical and inverted-conical thickeners, but also of upright-conical thickeners. In analyzing the thickening process, the equations used were, to the degree of detail needed for the present discussion, the same as used previously by the writer (Dixon, 1980). However, the clarification-capacity criterion, as given by Hazen (1904), and extended by Fitch (1956), was not considered. Primarily because of this, they reached, it must be said, incorrect conclusions.

Clarification criterion and under-capacity operation

In the most common situation, the feed volume fraction concentration, ϕ_o, is less than the gel or critical concentration, ϕ_g. For this case, Landman, White, and Buscall concluded that for a given underflow concentration, ϕ_u, there is only one possible solids throughput rate. That is, if the feed rate is reduced below the capacity limit, it is not possible to avoid producing a higher underflow concentration. However, this unexpected conclusion is the result of not taking into account the clarification process.

The Hazen/Fitch clarification criterion is not always properly understood, as has been discussed before by the writer (Dixon, 1985). It states that the overflow flux must be less than the free-settling velocity of the solids in the feed (assumed, as in the Landman, White, and Buscall treatment, to be the same for all the solids). This result is derived very simply from the requirement for the solids to settle from the overflow stream during the time that it takes a vertical element of the latter to flow from the inlet to the overflow outlet.

When working at less than full capacity, the solids have more time to leave the overflow stream than *necessary: that is*, the last solids (in a given vertical element) leave the bottom of the stream before the outlet position is reached. Thus, the solids leaving the overflow stream do not cover the whole of the cross section of the vessel.

If there is sufficient depth in the vessel, these solids will spread out to cover the whole cross-section before reaching the sludge level: that is, before compression effects are experienced. Since the total solids flow is the same at all levels at steady state, the solids concentration does not remain constant during this spreading process, but decreases to the value at which the free-settling flux (acting over the whole area) gives a solid flow equal to that obtained over part of the area at the feed concentration.

Most discussions are helped by reference to a graphical representation of the relationships. The Yoshioka et al. (1957), settling flux plot (Figure 1), is familiar to most workers with thickeners. The volumetric solids settling flux, ϕ_u, is the product of the concentration and the settling velocity, u, relative to the volume-average velocity of the suspension or sludge as a whole. In the absence of a compressive stress gradient (that is, when the concentration is not changing), the solids settle at their terminal velocity, u_t, a decreasing function of concentration. Thus, the terminal or free-settling flux, ϕu_t, is also a function of concentration, starting from zero at zero concentration, passing through a maximum, and approaching zero at volume fraction equal to unity, as illustrated in Figure 1.

The actual settling flux (as distinct from the terminal value, which is

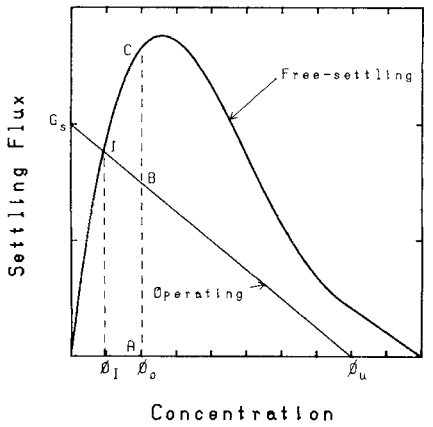


Figure 1. Flux plot including cylindrical operating line.

achieved only in the absence of a compressive stress or concentration gradient) is related to the concentration by material-balance considerations, since the flow rate of solids is the same at all levels of the thickener under steady-state conditions. By definition,

$$\phi u = G_s - cG_t = G_s\left[1 - \frac{\phi}{\phi_u}\right] = \frac{Q_p}{A}\left[1 - \frac{\phi}{\phi_u}\right] \quad (1)$$

where G_s is the solids flux (relative to the equipment), and G_t is the total flux (determined by the sludge pumping rate, also relative to the equipment), G_s equals the solids volumetric feed rate, Q_p, divided by the cross-sectional area, A. For thickeners of nonconstant area, while Q_p is the same at all depths, G_s and G_t vary with depth because the area varies. For the simplest case of a constant-area thickener, G_s and G_t are constant through the thickener, so that Eq. 1 gives a linear relation between the settling flux and the concentration (the "operating line").

The Hazen/Fitch clarification criterion translates easily into the terms used on the flux plot: namely, the feed-concentration point on the operating line must lie below the free-settling line. That is,

$$\phi u < \phi u_t(\phi) \quad (2)$$

for $\phi = \phi_o$. Thus, for the operating and free-settling lines shown in Figure 1, with feed concentration as shown, the fraction of the area over which solids leave the overflow stream is AB/AC. If the clarification zone were fully loaded, B and C would coincide; but, for the throughput and underflow concentration shown, the free-settling velocity at the feed concentration is more than enough to remove the solids from the overflow.

The solids leaving the overflow stream, together with the liquid that does not overflow, enter the thickening zone, where both solids and liquid move downwards. If there were sufficient depth to allow the solids to spread uniformly over the cross-section before the sediment is reached, the concentration will be ϕ_I (less than ϕ_o), at the intersection of the operating and free-settling lines. This is the only concentration at which the free-settling flux matches the operating requirements. Experimental studies of lightly-loaded thickeners often have reported the occurrence of a top uniform zone at a concentration less than that of the feed. [If the solids stream leaving the overflow stream were to remain at the feed concentration during its spread over the cross-section, the solids velocity would decrease, since the solids flow is the same at all levels at steady state. The solids would, thus, be settling at less than their terminal velocity and so, in the absence of compression effects, subject to an accelerating force. This results in a decrease in the concentration (Dixon, 1977).]

In a conical thickener or other thickener in which the cross-sectional area is not constant, the analysis is more complicated, since the cross-sectional area at which a given concentration occurs is not generally known in advance. The operating line is not straight and can be obtained only from solution (usually numerically) of the differential equations governing the thickening process. However, the basic Hazen/Fitch criterion still applies. Inequality 2 must be satisfied throughout the clarification zone. On the Yoshioka plot, the operating line is not straight for nonconstant area, but the ϕ_o point must lie below the free-settling value for ϕ_o.

Thus, Landman, White, and Buscall, in concluding that for $\phi_o < \phi_c$ there is one and only one possible throughput rate, overlook the fact that in their Eq. 49 the area should be only the actual area A_o when the clarification zone is fully loaded. Their equation is equivalent to requiring coincidence of points B and C on Figure 1, or of ϕ_o with ϕ_I. However, at less than full load, the area occupied by solids at the feed concentration is less than the available area, A_o, by the same factor as Q_p is less than its maximum value. Thus, with the correct area used in place of A_o, Landman, White, and Buscall's equation is still satisfied even when the clarification zone is not fully loaded. There is nothing to stop a thickener producing a given underflow concentration when operating at less than the maximum throughput.

Upright-conical thickener capacity

Landman, White, and Buscall, having concluded that thickener operation is inflexible when $\phi_o < \phi_g$, concentrated their attention on the case where $\phi_o \geq \phi_g$: that is, where the solids enter the thickener already in compression. However, in view of the preceding discussion, there is no special need to distinguish between the two cases. When $\phi_o > \phi_g$ there will be the "top-plug" effect, where thickening cannot start until the yield stress at the feed concentration is reached. However, this is not central to the basic question.

Landman, White, and Buscall concluded that upright-conical thickeners can always achieve a desired value of ϕ_u. In effect, their reasoning is that, even if the area at the top is insufficient to produce thickening, as the solids progress downwards the area increases so that thickening will start at some level and continue until it reaches the bottom. In the case of cylindrical and inverted-conical thickeners, the area does not increase downwards, and so if thickening does not occur at the top it will not occur at any level.

This means that at the start of the thickening process the lefthand side of Ineq. 2 can be larger than the right: that is, the operating point can be above the free-settling point on the flux plot. If the thickening-zone equations are integrated with respect to depth, the concentration will initially decrease. However, since the area increases as the solids progress downwards, a point is reached at which Ineq. 2 becomes an equality, where the operating line intersects the free-settling line, and thereafter the solids concentration increases (having Ineq. 2 satisfied).

This is illustrated in Figure 2 for an upright cone of 60° included angle, using the sludge data employed in the writer's previously-reported study (1980). The solids throughput is 6 mL/s and underflow volume fraction 0.333. The critical volume fraction has been taken as 0.2, the top area chosen so that the operating flux is 20% above the free-settling flux at this

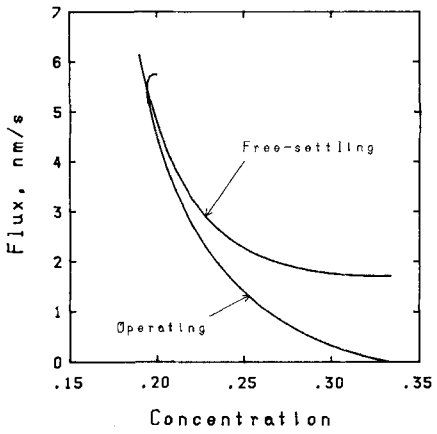


Figure 2. Impossible flux plot for upright-conical thickener.

concentration, and the operating line calculated, starting from that point.

The concentration initially decreases, as concluded by Landman, White, and Buscall. While the compression of flocs is expected to be inelastic, it is also expected that there will be at least a transient resistance to reexpansion. For the purpose of the illustration in Figure 2, the solids stress variation with concentration was taken as the same for compression and expansion. (The compressive stress decreases initially so that solid structure is effectively in tension.)

However, the operation shown in Figure 2 is impossible, no matter what the shape of the thickener, because the operating line cannot lie above the free-settling line. To satisfy the clarification requirements the area at the top must be sufficient so that Ineq. 2 is satisfied. Thus, it is not true that an upright cone can be as narrow as one likes at the top and still achieve any specified throughput and underflow concentration, provided that its depth is sufficient to provide enough area below. This denies the basis for the Landman, White, and Buscall claims about the superiority of upright conical thickeners.

Further, even if a thickener could operate this way, does it really show an upright cone to be superior to a cylinder or inverted cone? All that is involved is that insufficient area is provided at the top, but sufficient below. Why not simply have sufficient area all the way down? (The capital cost is probably no less for a conical thickener). The portion of the depth over which the area is too small involves concentration decrease, and so serves no useful purpose.

Sludge depth

While it cannot be accepted that upright cones are inherently superior and have no throughput limit for given ϕ_u, it still might be possible that they require smaller sludge depth. The comparisons given by Landman, White, and Buscall are on the basis of equal top areas, A_o. There is no question that an upright cone requires less depth than a cylinder or inverted cone on this basis. With the same area at the top in all cases, the upright cone provides more area at lower levels, so that there is less hydrodynamic support of the solids (the operating line is further below the free-settling line), and so thickening occurs more rapidly with respect to depth (assuming no sludge funneling).

However, one of the primary considerations in thickener selection is the ground area occupied. Thus, for upright cones and cylinders a more meaningful comparison is on the basis of equal bottom areas. In this case, the cylinder (assuming no funneling in both cases) will be superior. It provides the same area all the way up, while in the upright cone the area decreases with height, moving the operating line closer to the free-settling line and increasing the sludge depth required. An upright conical thickener has no thickening advantage over a cylindrical thickener and is no less likely to be affected by funneling, putting it out of contention.

Notation

A = cross-sectional area, L^2
A_o = value of A at the top (at which $\phi = \phi_o$)
G_s = solids volumetric flux, LT^{-1}
G_t = total volumetric flux, LT^{-1}
Q_p = solids volumetric flux rate, L^3T^{-1}
u = settling velocity, relative to the volume-average velocity, LT^{-1}
u_t = terminal settling velocity, with no compression effects, LT^{-1}
ϕ = solids volume-fraction concentration, dimensionless
ϕ_o = feed value of ϕ
ϕ_g = critical or gel value of ϕ
ϕ_I = ϕ at the intersection of operating and free-settling lines
ϕ_u = underflow value of u

Literature cited

Dixon, D. C., "Momentum-Balance Aspects of Free-Settling Theory: II. Continuous, Steady-State Thickening," *Sep. Sci.*, **12,** 193 (1977).

———, "Effect of Sludge Funneling in Gravity Thickeners," *AIChE J.*, **26,** 471 (1980).

———, "Capacity and Control of Clarifiers and Thickeners," *J. Water Pollut. Cont. Fed.*, **57,** 46 (1985).

Fitch, E. B., "Flow Path Effect on Sedimentation," *Sew. Ind. Wastes*, **28,** 1 (1956).

Hazen, A., "On Sedimentation," *Trans. ASCE*, **53,** 45 (1904).

Landman, K. A., L. R. White, and R. Buscall, "The Continuous-Flow Gravity Thickener: Steady State Behavior," *AIChE J.*, **34,** 239 (1988).

Yoshioka, N., Y. Hotta, S. Tanaka, S. Naito, and S. Tsugami, "Continuous Thickening of Homogeneous Flocculated Slurries," *Kagaku Kogaku*, **21,** 66 (1957).

D. Christopher Dixon
School of Chemical Engineering
and Industrial Chemistry
University of New South Wales
Kensington, N.S.W., Australia

Reply:

Dixon is correct in saying that we do not discuss the clarification criterion of Hazen. We did not wish to deal with the complications of the clarification zone and the overflow stream. In Dixon's terminology, we deal only with fully loaded thickeners. We assume that $\phi = 0$ in any overflow, so that in our problem the feed concentration is equal to the free-falling zone concentration ϕ_0. (This is illustrated in our Figure 2). Our analysis works for thickeners which are not fully loaded when ϕ_0 is taken to be the concentration at the start of the free-falling zone, but not the feed concentration. The clarification zone and overflow stream above the falling zone would have to be treated separately.

We agree with Dixon's comments on our statement that a diverging thickener can achieve any specified throughput and underflow concentration provided sufficient depth is provided. Our statements were naive and we appreciate his comments on this matter.

Dixon's other comments regarding economics of thickener design are similar to Fitch's and are dealt with in our previous reply. We have not considered this aspect of the problem.

L. R. White
K. A. Landman